Solution to Final Exam Paper B

1. [10 Points] A net force acts during the upward vertical motion of a 250-kg rocket, starting at the moment (*t*=0) when the fuel burned out and the rocket has upward speed of 120 m/s. Let *k*=0.65 kg/m. (a) Calculate the maximum height reached by the rocket. (b) Compare to free-flight conditions without air resistance (*k*=0).

Solution:

(a) We assume positive vertical *y* axis is upward and the maximum height the rocket reached is *h*.

Applying Newton’s second law to the rising rocket, we have

+2

Using the derivative chain role to replace the time t with vertical position y, we have

or

As we integrate from the initial position up to the highest point, we get

+2

+2

+1

which gives *h* = 302 m

(b) If there were no air resistance, a=-g. We find the maximum heigh from

+2

+1

which gives

2. [8 Points] The position of a 280-g object is given in meters by , where *t* is in seconds. Determine the net rate of work done on this object (a) at *t*=2.0s and (b) at *t*=4.0s. (c) What is the average net power input during the interval from *t*=0s to t=2.0s, and in the interval from t= 2.0s to 4.0s?

Solution:

+2

The rate of work is power

The applied force produce acceleration. So we find the velocity and acceleration as function of time:

+1

+1

Thus the rate of work is

1. t=2.0s

+1

1. t=4.0s

+1

1. Over a time interval, the average net power results in the change in kinetic energy:

The velocities at three occasions:

t=0s

t= 2.0s

t= 4.0s

From t=0s to t=2.0s, we have

+1

From t=2s to t=4.0s, we have

+1

3. [6 Points] Water strikes the turbine blades of a generator so that its rebounding velocity is 0.75 of its original magnitude and reversed in direction. If the flow rate is 60 kg/s and the original water speed is 10 m/s, what is the average force on the blades?

Solution:

+4

We find the average force on the water from

+1

According to Newton’s third law, the average force on the turbine blades is

+1

4. [10 Points] The radius of the roll of paper shown is 7.6 cm and its moment of inertia is . A force of 3.2 N is exerted on the end of the roll for 1.3 s, but the paper does not tear so it begins to unroll. A constant friction torque of 0. 11 m·N is exerted on the roll which gradually brings it to a stop. Assuming that the paper's thickness is negligible, calculate (a) the length of paper that unrolls during the time that the force is applied and (b) the length of paper that unrolls from the time the force ends to the time when the roll has stopped moving.



Solution:

We choose the clockwise direction as positive.

1. About the axis we have

+1

+1

which gives

The angle turned while the force is acting can be found from

+1

+1

The length of paper that unrolls during this time is

+1

1. With no force acting, we have

+1

+1

which gives

The initial angular velocity of this motion is

+1

The final angular velocity of this motion is 0. So

+1

which gives

+1

The length of paper that unrolls during this time is

5. [8 Points] Smoke particles in the air typically have masses of the order of 10-16 kg. The Brownian motion of rapid and irregular movement of these particles resulting from collisions with air molecules can be observed with a microscope. (a) Find the root-mean-square speed of Brownian motion for a particle with a mass of 3.00×10-16 kg in air at 300 K. (b) Would the root-mean-square speed be different if the particle were in hydrogen gas at the same temperature? Explain.

Solution:

Root-mean-square speed of a particle is decided by

+3

where

+2

1. No. The root-mean-square speed depends on the average kinetic energy of the particle. At this temperature T, hydrogen molecules would have larger than the typical air molecules but would have the same average kinetic energy with air molecules. The average kinetic energy of the smoke particles would be the same. So the root-mean-square speed of the smoke particles would be no different.

+3

6. [10 Points] When 5.30×104 J of heat are added to a gas enclosed in a cylinder fitted with a light frictionless piston maintained at atmospheric pressure, the volume is observed to increase from 2.2 m3 to 4.1 m3. Calculate (a) the work done by the gas, and (b) the change in internal energy of the gas. (c) Graph this process on a PV diagram.

Solution:

1. Because the pressure is constant, we find the work from

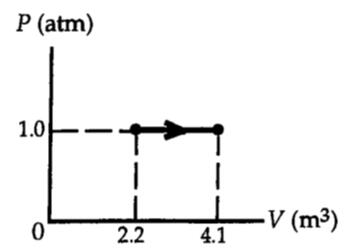
+1

+2

1. Using the first law of thermodynamics we find the change in internal energy

+1

+2

(c)

+4

7. [10 Points] A heat engine exhausts its heat at 360℃ and has a Carnot efficiency of 35 percent. What exhaust temperature would enable it to achieve a Carnot efficiency of 50 percent?

Solution:

From Carnot efficiency of 35 percent,

+2

+3

we find

Since high temperature reservoir does not change, the new efficiency is

+2

+3

which gives

8. [10 Points] A pendulum consists of a tiny bob of mass *M* and a uniform cord of mass *m* and length *L*. (a) Determine a formula for the period of this complex pendulum. (b) Is the period of this complex pendulum larger or less than that of a simple pendulum (the mass of cord is negligible)? (c) If what would be the fractional error if you take it as a simple pendulum?



Solution:

1. The rotational inertia of the system about the suspension point is

+2

The distance of the center of mass of the system from the suspension point is

+2

So the period for small angle oscillations is

+2

1. For simple pendulum

Since

+2

If we were to take the system as a simple pendulum, the factional error would be

+1

+1

9. [10 Points] A water wave traveling in a straight line on a lake is described by the equation:

*y(x,t)*=(3.75 cm)cos(0.450 cm-1*x*+5.40s-1*t*)

Where *y* is the displacement perpendicular to the undisturbed surface of the lake. (a) How much time does it take for one complete wave pattern to go past a fisherman in a boat at anchor, and what horizontal distance does the wave crest travel in that time? (b) What are the number of waves per second that pass the fisherman? (c) How fast does a wave crest travel past the fisherman, and what is the maximum speed of his cork floater as the wave causes it to bob up and down?

Solution:

Comparing given wave function

*y(x,t)*=(3.75 cm)cos(0.450 cm-1*x*+5.40s-1*t*)

with general form of the wave function for a wave traveling in -*x* direction

We find A=3.75cm, ,

(a) The time for one complete cycle to pass a point is the period *T*.

+2

The horizontal distance the wave crest travels in one period is wavelength .

+2

(b) The number of waves that pass the fisherman per second is frequency .

+2

(c) The speed of the wave crest is the speed of the wave

+2

The maximum speed of the particles in water is

+2

10. [6 Points] What is the minimum non-zero thickness for the air layer between two flat glass surfaces if the glass Is to appear dark when 640-nm light is incident normally? What if the glass is to appear bright?

Solution:

0

*t*

The reflected light wave has no phase shift at the top surface of the air layer, but the reflected light wave has a phase shift at the bottom surface of the air layer.

So for normal incident light, the optical path difference for the two reflected light beams from the top and bottom surfaces is

For destructive interference

+2

which gives the thickness of the air layer

m=0,1,2,….

Therefore the minimum non-zero thickness is

+1

For constructive interference

+2

which gives the thickness of the air layer

m=1, 2,….

The minimum thickness is

+1

11. [6 Points] If a double-slit pattern contains exactly seven fringes in the central diffraction peak, what is the relation between the slit width and separation?

Solution:

If the central diffraction peak contains 7 fringes, there will be 3 on each side of the center. Thus the fourth maximum of the double-slit must coincide with the first minimum of the diffraction pattern.

The maxima of the double-slit pattern are given by

+2

=0, ±1, ±2,…

The minima of the single-slit pattern are given by

+2

Thus we have

or

+2

12. [6 Points] (a) How far away can a human eye distinguish two car headlights 2.0 m apart? Consider only diffraction effects and assume an eye diameter of 5.0 mm and a wavelength of 500 nm. (b) What is the minimum angular separation an eye could resolve when viewing two stars, considering only diffraction effects?

Resolution:

(a) The resolution of the eye is

+2

The maximum distance is

which gives

+2

(b) The angular separation is the resolution:

+2